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DETERMINATION OF THE DRAG ON OSCILLATING PLATES IN A FLUID*

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Using an approximate approach /1/, methods of determining the vortex drag on plates undergoing harmonic oscillations in an incompressible fluid are considered. By means of this approach, the problem can be reduced to determining the velocity intensity coefficients (VIC's) on the edges of the plates and computing a certain integral over the boundary contour. Mathematically, the VIC's are analogous to the stress intensity coefficients (SIC's) /2/ in destruction mechanics. The most important exact solutions and closed expressions for the VIC's are presented for the planar and the spatial problems. To obtain numerical solutions, a version of the direct boundary-element method (BEM) is developed. Examples of applications of the finite-element method (FEM) and the BEM to specific problems are given. Methods for improving the accuracy of the numerical solutions are proposed. The results of experimental investigations are presented and compared with the computations.

1. Formulation of the problem. Consider the oscillations of a plate in an incompressible fluid at rest at long distances. We introduce the following notation: R is the characteristic linear dimension of the plate, v_0 and ω are the characteristic velocity amplitude and oscillation frequency of the plate, ρ and ν are the density and the kinematic viscosity of the fluid, and $\mathrm{Re} = v_0R/\nu$ and $\mathrm{Sh} = R\omega/v_0$ are the Reynolds and Strouhal numbers. We shall assume that the condition

$$\mathrm{Re}^{-1/2} \ll \mathrm{Sh}^{-1/3} \ll 1 \tag{1.1}$$

is satisfied.

The condition establishes a relation between the orders of magnitude of the thickness of the oscillating boundary layer, the dimensions of the eddy domain in the vicinity of the sharp edges, and the dimensions of the plate /1/. Outside small domains of essential eddies the motion of the fluid will be assumed to be a potential one.

We represent the velocity potential of the fluid in the form $\Phi(\mathbf{r}, t) = \varphi(\mathbf{r}) \cos \omega t$, where $\mathbf{r}$ is the position vector of a point and t is the time.

We have the boundary condition $\partial\varphi/\partial n = \mp v_n(\mathbf{r})$ on the surface of the plate. The "minus" and "plus" signs correspond to the "positive" and "negative" sides of the plate, which is assumed to be infinitely thin and $\mathbf{n}$ denotes the outer normal unit vector.

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Under the above assumptions, the drag coefficient can be defined as follows /1/:

$$c_D = k\,\mathrm{Sh}^{1/3}, \quad k = {}^3/_4 B\,(\mathrm{Re})\,I\,(v_n) \tag{1.2}$$

$$(v_n) = \oint\limits_l \left(\frac{K_v^2}{Rv_0^2}\right)^{4/3} d\,\frac{l}{R} \tag{1.3}$$

In general, this definition corresponds to the generalized drag force $Q = -{}^1/_2 c_D \rho v_0^2 R^2 \,|\cos\omega t\,|\cos\omega t$ acting on the plate. In (1.3) l denotes the boundary countour of the plate and the VIC can be computed as follows:

$$K_v(l) = -\lim_{r\to 0}(2\pi r)^{1/2}\,\partial\varphi/\partial n = \lim_{r\to 0}(2r/\pi)^{-1/2}\,\varphi_+ \tag{1.4}$$

where r is the distance measured in the tangent plane along the outer and the inner normal vector to l in the former and the latter formula, respectively, and where $\mathbf{n}$ denotes the chosen positive normal unit vector to the tangent plane. In (1.2) $B\,(\mathrm{Re}) \approx 2 + O\,(\mathrm{Re}^{-1/2})$ for large Reynolds numbers /1/. Other experimental data /3, 4/ and computations by the method of discrete vortices confirm (1.2), but do not enable one to improve that relation.

Since, according to (1.1), the domains of essential eddying are small, the problem of determining the form of the oscillations of the plate can be solved independently to a first approximation if the plate is elastic. We shall assume that this problem is solved and $v_n(\mathbf{r})$ is known. Then, to determine the drag, we have to find the harmonic function φ from the given boundary condition and compute the VIC on the contour of the plate. Exact and approximate methods for solving that problem will be considered below.

2. ***Exact solutions of the plane problem.*** We denote by $w(z) = u - iv$ the velocity function of the absolute flow of the fluid in the complex plane $z = x + iy$.

We consider the oscillations of a flat plate for a given curve representing the velocity distribution $v_n(x)$ on the plate (Fig.1a). Applying a Cauchy-type integral, we get

$$w(z) = -\frac{1}{\pi g(z)}\int_{-a}^{a}\frac{g(\tau)\,v_n(\tau)}{\tau - z}\,d\tau \tag{2.1}$$

where $g(z) = (z^2 - a^2)^{1/2}$ is the analytic branch determined by the condition $g(z)/z \to 1$ as $|z| \to \infty$. Computing the VIC by means of (1.4), we find that

$$K_v(\pm a) = \frac{1}{(\pi a)^{1/2}}\int_{-a}^{a}\left(\frac{a \pm \tau}{a \mp \tau}\right)^{1/2} v_n(\tau)\,d\tau \tag{2.2}$$

For translational oscillations with $v_n(x) = v_0$ we find from (2.2) that $K_v(\pm a) = v_0(\pi a)^{1/2}$. In the case of rotation about the central axis with $v_n(x) = v_0 x/a$ we have $K_v(\pm a) = \pm{}^1/_2 v_0(\pi a)^{1/2}$. Since the problem is linear, it follows that, using those two expressions, one can write the VIC as a superposition for arbitrary oscillations of the plate as a rigid body. In particular, for angular oscillations about the left edge with $v_n(x) = v_0(1 + x/a)$, we have $K_v(a) = {}^3/_2 v_0(\pi a)^{1/2}$ and $K_v(-a) = {}^1/_2 v_0(\pi a)^{1/2}$. For elastic oscillations of the form $v_n(x) = v_0(x/a)^2$ we find from (2.2) that $K_v(\pm a) = {}^1/_2 v_0(\pi a)^{1/2}$.

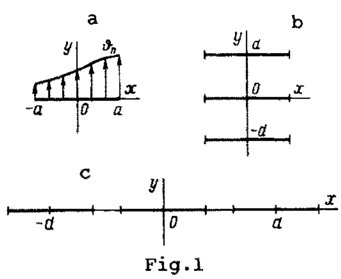

Fig.1

To determine the drag per unit length of the plate in the case of the plane problem, we shall replace the integral (1.3) by the sum $I(v_n) = \Sigma\,[K_v^2/(Rv_0^2)]^{4/3}$ over all sharp edges. For example, in the case of rotation about the central axis, $I(v_n) = 2(\pi/4)^{4/3}$, $k = 3(\pi/4)^{4/3} = 2.17$ in (1.2), and the drag moment

$$M = -{}^1/_2 c_D\,\rho\theta^{\cdot\,2} a^3\,|\cos\omega t\,|\cos\omega t, \quad \theta^{\cdot} = v_0/a, \quad R = a$$

The method for taking into account any singularities in the fluid (sources, vortices, etc.) is well-known /5/. The problem can be solved independently when the plate is assumed to be at rest. In this case, if there is a source q and a vortex γ at z_0 then

$$w(z) = \frac{1}{2}\left(\frac{\beta}{z - z_0} - \frac{\bar\beta}{z - \bar z_0}\right)\frac{1}{(z^2 - a^2)^{1/2}} + \frac{1}{2}\left(\frac{\alpha}{z - z_0} + \frac{\bar\alpha}{z - \bar z_0}\right)$$

$$\alpha = (q + i\gamma)/(2\pi), \quad \beta = \alpha\,(z_0^2 - a^2)^{1/2}$$

Hence, computing the limit (1.4), we get

$$K_v(\pm a) = (\pi a)^{1/2}\,\mathrm{Im}\,[\alpha\,(z_0^2 - a^2)^{1/2}/(a \mp z_0)] \tag{2.3}$$

Expression (2.3) can be used as Green's function to find other solutions, in particular to find (2.2).

We shall consider a periodic system of parallel plates undergoing the same form of oscillations (Fig.1b). The function $\zeta = f(z) = \mathrm{th}\,(\pi z/d)$ has period id and transforms the strip $-d/2 < y < d/2$ in the z-plane into the whole ζ-plane with a cut along the real axis, i.e., it reduces the problem to that considered above. Using (2.1) and taking the properties of conformal mappings into account, we find that

$$w(z) = -\frac{f'(z)}{\pi g(\zeta(z))}\int_{-a}^{a}\frac{g(\tau(t))\,v_n(\tau(t))}{\tau(t) - \zeta(z)}\,dt \tag{2.4}$$

$$g(\zeta) = [\zeta^2 - \mathrm{th}^2\,(\pi a/d)]^{1/2}, \quad \tau(t) = \mathrm{th}\,(\pi t/d)$$

Evaluating the limit (1.4), we get

$$K_v(\pm a) = \frac{1}{[(d/2)\,\mathrm{sh}\,(2\pi a/d)]^{1/2}}\int_{-a}^{a}\left[\frac{\mathrm{th}\,(\pi a/d) \pm \mathrm{th}\,(\pi t/d)}{\mathrm{th}\,(\pi a/d) \mp \mathrm{th}\,(\pi t/d)}\right]^{1/2} v_n(t)\,dt \tag{2.5}$$

If $a/d \ll 1$, then (2.5) can be reduced to (2.2). In the case of translational oscillations with $v_n(x) = v_0$ the integral in (2.5) can be computed using the theory of residues:

$$K_v(\pm a) = v_0\,[d\,\mathrm{th}\,(\pi a/d)]^{1/2} \tag{2.6}$$

Formula (1.2) for determining the drag is applicable in this case provided the condition (1.1) is satisfied not only for $R = a$, but also for $R = d$. For small d/a the lower limit of admissible Strouhal numbers should be increased.

We consider a periodic system of coplanar plates undergoing the same form of oscillations (Fig.1c). We use the function $\zeta = f(z) = i\,\mathrm{tg}\,(\pi z/d)$ of period d, which transforms the strip $-d/2 < x < d/2$ in the z-plane into the whole ζ-plane with a cut along the real axis. In the case in question the results can be obtained from (2.4)-(2.6) if the hyperbolic functions are replaced by the corresponding trigonometric functions. Then, by analogy with (2.6), $K_v(\pm a) = v_0\,[d\,\mathrm{tg}\,(\pi a/d)]^{1/2}$ for $v_n(x) = v_0$ and it becomes obvious that the relation also holds for a plate oscillating in a flat channel $-d/2 < x < d/2$ with rigid walls. The drag on the plate in the channel exceeds that in an unbounded fluid. For small gaps, (1.2) is applicable if (1.1) is also satisfied for $R = d/2 - a$.

As an application we consider the oscillations of a plane with ribs covered by an unbounded fluid later. It is assumed that the ribs are uniformly distributed and perpendicular to the plane, and they have the same height a. It is obvious that the VIC can be determined by means of (2.6). The energy spent on forming vortices during a single oscillation period per unit length L and unit width l of the plate can be written in the form /1/

$$E/(Ll) = B\,(\mathrm{Re})\mathrm{Sh}^{-1/3}\rho v_0^2 a\,[(a/d)/(a v_0^2/K_v^2)^{-4/3}] \tag{2.7}$$

We find from (2.7) that, for a given height of the ribs, the drag will reach its maximum for $d/a = 1.93$. It turns out that the dependence of the drag on the distance between the ribs is weak. At the end-points of the interval $1 < d/a < 3$ it is only 10% less than the maximum.

Using the Keldysh-Sedov formula /5/, one can obtain closed expressions for the VIC's by considering the oscillations of a system consisting of a number of coplanar plates.

Because of the mathematical analogy between plane problems of hydrodynamics and the compound displacement in the theory of elasticity noted in /6/, an analogy between the VIC's and the SIC K_{III} for the longitudinal displacement can be found /7/.

3. Exact solutions of the spatial problem. We consider the oscillations of a flat plate in an unbounded fluid. We introduce an xyz orthogonal coordinate system such that the y and z axes lie in the plane of the plate. It is obvious that the velocity potential is antisymmetrical about the $x = 0$ plane, and so the problem can be stated for a half-space: for $x = 0, \partial\varphi/\partial x = v_n(y, z)$ in the domain S inside the contour of the plate and $\varphi = 0$ outside S. Exact solutions of the problem are available for circular or elliptic domains S. The general solution for a circle of radius a has the form /8, 9/

$$v(0, y, z) = -\frac{1}{\pi^2 (y^2 + z^2 - a^2)^{1/2}}\iint_S \frac{(a^2 - \xi^2 - \eta^2)^{1/2}\,v_n(\xi, \eta)\,d\xi\,d\eta}{(y - \xi)^2 + (z - \eta)^2}$$

$$y^2 + z^2 > a^2, \quad \xi^2 + \eta^2 \leqslant a^2$$

Evaluating the limit (1.4), we get

$$K_v(\theta) = \frac{1}{\pi(\pi a)^{1/2}} \iint\limits_S \frac{(a^2 - \xi^2 - \eta^2)^{1/2} v_n(\xi, \eta)\, d\xi\, d\eta}{(a\cos\theta - \xi)^2 + (a\sin\theta - \eta)^2} \tag{3.1}$$

where θ is the polar angle. For translational oscillations with $v_n(y, z) = v_0$ and angular oscillations about the z-axis with $v_n(y, z) = v_0 y/a$, it follows from (3.1) that /7/

$$K_v(\theta) = 2v_0 (a/\pi)^{1/2}, \quad K_v(\theta) = {}^4/_3 v_0 (a/\pi)^{1/2} \cos\theta \tag{3.2}$$

respectively. We can now write down the expression for the VIC for any form of oscillations of a circular plate as a rigid body.

For oscillations about the diameter, using K_v from (3.2) and evaluating the integral (1.3), we find that $I(v_n) = 1.31$. The resulting drag moment will be

$$M = -{}^1/_2 c_D \rho \theta^{\cdot 2} a^5 |\cos \omega t| \cos \omega t$$

$$c_D = 0.92\mathrm{Sh}^{1/3}, \quad \theta^{\cdot} = v_0/a$$

The above problem for a harmonic function in a half-space plays an important role in crack mechanics /9/. On the boundary $x = 0$ of the half-space the velocity potential corresponds to the x-displacement to within a multiplicative constant and the normal component $v_n(y, z)$ of the velocity corresponds to the normal load $p(y, z)$ on the surface of the crack. The correspondence is no longer valid inside the media, but the breakaway SIC K_I can also be computed from (1.4), and so $K_v = K_\mathrm{I}$ for $p(y, z) = -v_n(y, z)$. This makes it possible to use the solutions for a flat crack or a system of coplanar cracks. Some closed expressions for the SIC K_I for an elliptic domain S are given in /7/. As a result of the established correspondence, the expressions are also valid for the VIC. In the remaining cases one has to resort to approximate or purely numerical methods.

4. Application of the BEM. Using the method of weighted residues /10/, we can write

$$\int\limits_Q w\nabla^2\varphi\, dQ = \int\limits_{\Gamma_1} \left(\frac{\partial\varphi}{\partial n} - v_n\right) w\, d\Gamma - \int\limits_{\Gamma_2} (\varphi - f) \frac{\partial w}{\partial n}\, d\Gamma \tag{4.1}$$

where Q is the domain occupied by the fluid, Γ_1 is the boundary on which the normal component $\partial\varphi/\partial n = v_n$ of the velocity is given, Γ_2 is the boundary on which the values of the potential $\varphi = f$ are given, and n is the outer normal unit vector to $\Gamma = \Gamma_1 + \Gamma_2$. As the weight function w in (4.1) we take the fundamental solution of the equation

$$\nabla^2 w_{ei} = \partial\delta_i/\partial e \tag{4.2}$$

on the right-hand side of which there is the derivative of the Dirac δ function at the point i in the direction e. Applying the integration-by-parts formula twice to the left-hand side of (4.1) and using (4.2), we get

$$v_{ei} + \int\limits_{\gamma_2} \frac{\partial\varphi}{\partial n} w_{ei}\, d\Gamma = \int\limits_{\gamma_1} (\varphi_+ - \varphi_-) \frac{\partial w_{ei}}{\partial n}\, d\Gamma + \int\limits_{\gamma_2} \varphi \frac{\partial w_{ei}}{\partial n}\, d\Gamma \tag{4.3}$$

Here γ_1 is one of the sides of that part of Γ both sides of which, namely, the "positive" side $\gamma_1^+ = \gamma_1$ and the 'negative' side γ_1^-, interact with the fluid. φ_+ and φ_- are the values of the potential on γ_1^+ and γ_1^- respectively. It is assumed that there are rigid bodies with surface γ_2 and infinitely thin plates with surface $\gamma_1^+ + \gamma_1^-$ in the fluid and $\Gamma = \gamma_1^+ + \gamma_1^- + \gamma_2$. The integral over $\gamma_1^+ + \gamma_1^-$ on the left-hand side of (4.3) vanishes, since the normal velocity is continuous. If all the boundary conditions are known, the velocities v_{ei} of the fluid can be determined from (4.3) (the former index indicates the direction, while the latter indicates a point within the domain).

The system of equations of the BEM can be obtained as follows. The boundary $\gamma_1 + \gamma_2$ of the domain is divided into N elements. In the simplest case the functions $u = (\varphi_+ - \varphi_-$ on γ_1 and φ on $\gamma_2)$ and $\partial\varphi/\partial n$ are assumed to be constant and equal to u_j and v_{nj} on the j-th boundary element (BE), i.e., the piecewise constant approximation is used. The weight function w_{nj} at the centre of the j-th element, n being the outer normal vector to the boundary, can be defined in accordance with (4.2), and Eq.(4.3) can be written for each BE. As a result, we arrive at the closed system of linear algebraic equations

$$v_{ni} + \sum_{j=1}^{M} v_{nj} \int\limits_{\Gamma_j} w_{ni}\, d\Gamma + \sum_{j=1}^{N} u_j \int\limits_{\Gamma_j} \frac{\partial w_{ni}}{\partial n}\, d\Gamma \tag{4.4}$$

since N of the $2N$ quantities u_j and $v_{nj}(j = 1, 2, \ldots, N)$ are determined by the boundary conditions.

Here Γ_j is the boundary of the j-th element and the BE's are numbered in such a way that the first M elements belong to γ_2 and the other ones to γ_1. The singular integrals in (4.4) can be understood in the sense of the main value and can be defined as the limits as i approaches the boundary along the normal line from the interior of the domain occupied by the fluid.

The approximation adopted above does not ensure that the solution is accurate near the sharp edges. Therefore, to obtain admissible values of the VIC's, we can use their connection with the kinetic energy /1/, which can be expressed in the form

$$v_0^2 \partial\mu/\partial n = \rho \int K_v^2 dl \tag{4.5}$$

Evaluating the generalized associated mass μ for a plate whose dimensions along the normal line to the contour in the tangent plane are changed by Δn, we obtain an estimate for $\partial\mu/\partial n$ and the integral in (4.5). It turns out that the estimate is not worse than the value of μ. Moreover, the error in computing μ can be substantially reduced by applying the non-linear Shanks transformation /11, 12/

$$\mu = (\mu_1\mu_3 - \mu_2^2)/(\mu_1 + \mu_3 - 2\mu_2) \tag{4.6}$$

if it is possible to carry out the computations to obtain a few values μ_1, μ_2 and μ_3 on geometrically similar grids, starting from a crude division into BE's.

As in the case of the plane problem with $K_v(l) = \mathrm{const}$, only one parameter can be determined from (4.5). Even though the numerical values of the VIC's determined from (1.4) are characterized by a large error, they give a true picture of the changes of $K_v(l)$ along the contour of the plate and reduce the problem to a single parameter, which can be found from (4.5). Such a procedure can be applied to a part of the contour, e.g., to one of the sides of a rectangular plate.

We remark that in a number of cases (4.5) provides a convenient method for the analytic determination of the VIC's. We shall demonstrate this point using as an example the oscillations of a circular plate about its diameter, which were considered above. In this case it is obvious that $K_v(\theta) = C\cos\theta$ and the associated moment of inertia of the fluid $J = (16/45)\rho a^5$ /13/. Substituting J and K_v into (4.5), we find that $C = {}^4/_3\theta^{\cdot}(a^3/\pi)^{1/2}$.

5. *Numerical solution of plane problems.* The FEM and BEM applied to bounded domains occupied by a fluid are equally effective. In the case of the FEM the matrices have large dimensions, but they are band matrices. For the BEM, reduction to a one-dimensional problem is feasible, the matrices have lower dimensions, but they are completely filled. For an unbounded domain occupied by the fluid, it is better to use the BEM. Since it is well-known how to apply the FEM in hydrodynamics /14/, in the sequel we shall consider only the algorithm of the BEM.

We introduce orthogonal coordinate system xy and $\xi\eta$ connected with the i-th and j-th BE's, which are straight intervals approximating the contour l (Fig.2a). $\mathbf{R}$ will denote the position vector relative to the centre of the i-th BE, θ will be the angle between $\mathbf{R}$ and the η axis, and ψ will be the angle between the x and η axes. From (4.2) we find that

$$w_{ni} = -\frac{1}{2\pi}\frac{x}{R^2}, \quad \frac{\partial w_{ni}}{\partial n} = -\frac{1}{2\pi}\frac{\sin(\psi - 2\theta)}{R^2} \quad \text{on } \Gamma_j \tag{5.1}$$

The integrals of the functions (5.1) appearing in (4.4) can be computed analytically:

$$\int_{\Gamma_j} w_{ni}\, d\Gamma = \frac{\sin\psi}{2\pi}(\theta_2 - \theta_1) - \frac{\cos\psi}{2\pi}\ln\frac{R_2}{R_1} \tag{5.2}$$

$$\int_{\Gamma_j} \frac{\partial w_{ni}}{\partial n}\, d\Gamma = \frac{\sin(\psi - \theta_1 - \theta_2)}{2\pi}\left(\frac{\cos\theta_1}{R_2} - \frac{\cos\theta_2}{R_1}\right)$$

The notation used in (5.1) and (5.2) is explained in Fig.2, Eqs.(4.4) and relation (5.2) provide a very efficient algorithm for solving the plane problem for an arbitrary domain and can easily be turned into computer programs.

As an example we present the computation for the oscillations of a plate with a cut of the form of the arc of a semicircle in the plane of symmetry (Fig.2b) in an unbounded fluid. Taking the symmetry into account, we divided a half of the arc into BE's. In the computation of the VIC from (4.5) the variation of the angle half-measure of the arc was ±0.01. The results are presented in Table 1, in which the corrections following from formula (4.6) are given in brackets. Comparing the results with the analytic solution $\mu/(\rho a^2) = 3\pi/2$ and $K_v^2/(av_0^2) = \pi/2$, we can see that, using a small number of BE's, one can obtain sufficiently accurate values, which is important when solving spatial problems.

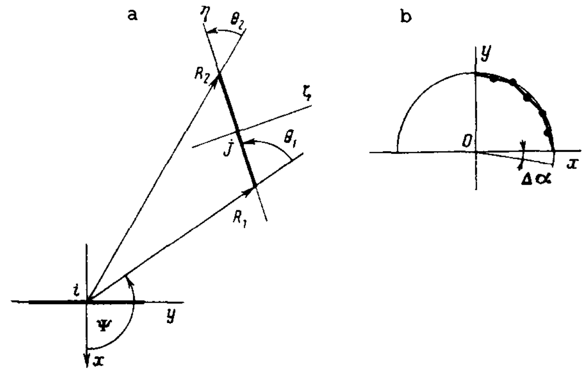

Fig.2

Let us consider the problem of the damping of the oscillations about the longitudinal axis of a circular cylindrical container with a flat bottom filled with a fluid in the presence of uniformly distributed radial dividing walls (Fig.3). In /15/, in which the general problem of the motion of a rigid body with a cavity containing a fluid of small viscosity was considered, the walls of the cavity were assumed to be fairly smooth. The concept of a boundary layer is not applicable in the vicinity of a sharp edge. If the characteristic dimensions of the cavity are of the order of 1 m, the order of the vortex damping connected with the flow past the sharp edges does not exceed the damping caused by a boundary layer near the walls.

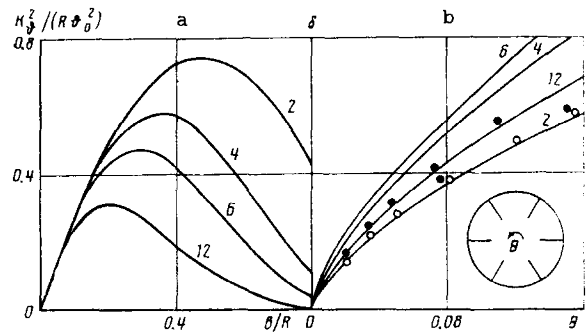

Fig.3

Let R be the radius of the cavity, b the width of the dividing walls, H the depth of the fluid and θ the amplitude of the angle of rotation about the longitudinal axis. Setting $v_0 = R\omega\theta$ and taking into account that the flow past any of the walls has the same character, we can represent the dependence of the decrement of the oscillations in the form

$$\delta = k\theta^{2/3}, \quad k = \frac{NB(\mathrm{Re})\rho H R^4}{I_0 + IH}\left(\frac{K_\vartheta^2}{Rv_0^2}\right)^{4/3} \tag{5.3}$$

where N is the number of walls, I is the associated moment of inertia of unit layer of the fluid, and I_0 is the moment of inertia of the container without the fluid.

We shall present the solution of the problem by the FEM. Taking into account the symmetry of the cavity, we carried out the computations for a single sector. Up to 2000 linear triangular elements were used. The resulting values of I were in agreement with those given in /16, 17/. The results of determining the VIC's are represented in Fig.3a by a number of lines. The digits written next to those lines indicate the number of dividing walls. Using these relations in (5.3), one can choose the width and the number of walls necessary to ensure the required damping.

Let us present a comparison with the experimental data obtained by Churilov for a cavity with $R = 0.35$m, $b/R = 0.3$, and $I_0/(\rho H R^4) = 0.884$. In this case the theoretical relations (5.3) are represented in Fig.3b by a number of lines. The maximum damping was obtained in the case of 6 walls. In the case of 4 and 8 walls the damping is practically the same and is represented by a single line. The experimental results are indicated by the circles in the case of 2 walls and by the dark circles in the case of 4 walls. The agreement between the computational and experimental data is good in the case of 2 walls. For a cavity with 4 dividing walls the computational line lies much higher, which may be connected with the influence of the free surface of the fluid as well as large-scale vortices appearing for large amplitudes of the oscillations. If the width of the dividing walls is small, $b/R < 0.1$, the results of the solution of the problem concerning the oscillations of a plane with ribs covered by an unbounded fluid layer can be used.

Table 1

Number of BE's	4	8	16	32
$\mu/(\rho a^2)$	4.958	4.850	4.785 (4.689)	4.749 (4.705)
$K_v^2/(a v_0^2)$	1.448	1.525	1.551 (1.565)	1.562 (1.571)

Table 2

a/b	1	2	3	4
c_m, BEM	0.578	0.756	0.831	0.871
c_m, /18/	0.557	0.742	0.828	0.873
k_x, BEM	4.64	5.30	5.72	5.98

6. Solution of the spatial problem by the BEM. We introduce orthogonal coordinate systems xyz and $\xi\eta\zeta$ with origins at the centres of the i-th and j-th BE's such that the x and ξ axes are directed along the corresponding normal lines. It follows from (4.2) that

$$w_{ni} = -\frac{1}{4\pi}\frac{x}{R^3}, \quad \frac{\partial w_{ni}}{\partial n} = \nabla w_{ni}\frac{\partial \mathbf{R}}{\partial \xi} \quad \text{on } \Gamma_j$$

Below we shall restrict ourselves to considering the oscillations of a flat plate in an unbounded fluid. In this case the surface γ_2 is not present ($M = 0$ in (4.4)) and the integral

$$\int_{\Gamma_j} \frac{\partial w_{ni}}{\partial n}\, d\Gamma = \frac{1}{4\pi}\oint \frac{d\theta}{r}$$

for triangular and rectangular piecewise-constant BE's can be easily evaluated in an analytic form in the cylindrical coordinate system $xr\theta$ connected with the point i.

For a rectangular plate with sides of length $2a$ and $2b$ ($a \geqslant b$) we studied the translational oscillations perpendicular to the plane of the plate. The computations were carried out for one quarter of the plate. Up to 256 BE's were used. Relations (4.5) and (4.6) were used to determine various quantities.

For the sake of presenting the results we find it convenient to introduce the associated mass coefficient $c_m = \mu/(2\pi\rho b^2 a)$ and the drag coefficient $c_x = c_D R^2/S = k_x \mathrm{Sh}^{1/2}$ ($\mathrm{Sh} = b\omega/v_0$) per unit area S. Here, according to (1.2), $k_x = kR^2/S$.

We list in Table 2 the values of c_m obtained by the BEM and from an empirical formula /18/ that approximates the experimental data, as well as the values of k_x obtained by the BEM. The computational and experimental values of c_m are in good agreement. For a plate of infinite length ($a \to \infty$), $c_m = 1$ and $k_x = 6.9$. The shorter the plate the lower the values of c_m and k_x, and, for a square plate, the values reach the level of 58% and 66% of the limiting values, respectively. The dependence of $K_v(l)$ on the lengths of the sides of the plate can be found in /7, 19/ if one takes into account that $K_v/v_0 = -K_I/p$.

We studied the oscillations (rotations) of the plate about the middle line parallel to a. Up to 256 BE's per one quarter of the plate were used. The dependence of $K_v(l)$ along the sides of a square plate obtained by the BEM is shown as an example in Fig.4a. The solid line corresponds to the parallel rotation axis and the broken line to the perpendicular rotation axis; l is measured from the middle of the corresponding side. For a plate of infinite length we obtained $K_{v\infty} = 1/2\,(\pi b)^{1/2}$.

We express the moment of the drag force in the form

$$M = -{}^1\!/_2 c_\theta \rho \theta^{\cdot 2} S b^3 \,|\cos \omega t|\, \cos \omega t$$

$\theta^{\cdot}$ being the angular velocity amplitude. Here $c_\theta = k_\theta \mathrm{Sh}^{1/3}$, $k_\theta = kb^2/S$, and the coefficient k can be determined from formulae (1.2) and (1.3), in which we set $R = b$, $v_0 = b\omega\theta = b\theta^{\cdot}$ and, consequently, $\mathrm{Sh}^{-1} = \theta$. We can express the associated mass coefficient in terms of the associated moment of intertia J_f of the fluid by the formula

$$c_m = 8J_f/(\pi\rho b^4 a).$$

Below we given c_m and k_θ for some values of the ratio of the sides of the plate:

a/b	0,25	0,5	0	2	4
c_m	0,455	0,653	0.810	0,898	0.945
k_θ	0.830	0.884	0.960	1.012	1 043

For a plate of infinite length, $J_f = \pi\rho b^4/8$ and $k = 2.174$, which corresponds to $c_m = 1$ and $k_\theta = 1.087$.

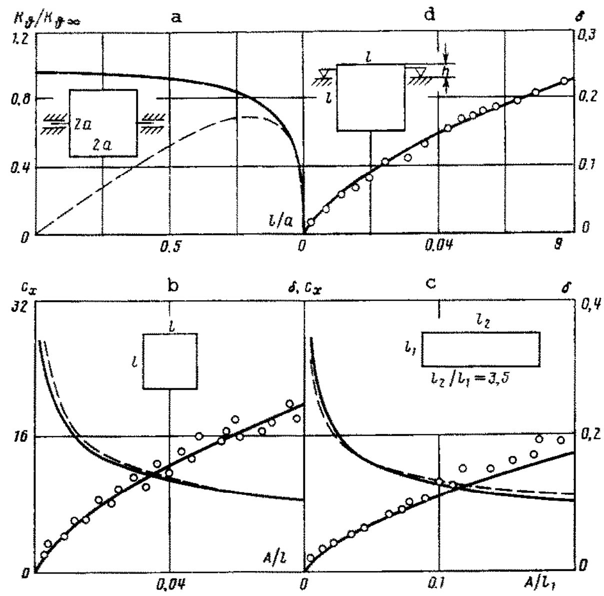

Fig.4

7. Experimental investigations of the drag for the oscillations of rectangular plates. Experimental tests were carried out for rectangular plates in air. A square plate with sides $l = 1.35$ m and a rectangular plate with sides $l_1 = 0.5$ m and $l_2 = 1.85$ m made from AmG-6 sheet of thickness 1.5 mm were chosen. To eliminate bending deformations, the plates were strengthened by a 7 mm high trapezoidal profile, but the regions up to 90-100 mm apart from the edges were left free. The plates were hung vertically in such a way as to minimize the influence of the friction of the suspension, which was eliminated anyway at a later stage by carrying out special additional tests. By changing the springs, we were able to change the oscillation frequencies in different series of tests from 0.39 Hz to 1.3 Hz. By measuring the force by a dynamometer attached to one of the springs, we registered free fading oscillations of the plate in suspension. From the oscilloscope record we worked out the dependence of the logarithmic damping decrement on the amplitude of the oscillations.

The dependence of the decrement on the amplitude represented using the logarithmic scale indicates that the experimental data lie on a straight line, except for amplitudes less than 2-4 times the thickness of the plate, for which the error of the measurements becomes large. This suggests that the decrement depends exponentially on the amplitude. Values between 0.65 and 0.77 were obtained for the exponent in the case where its theoretical value was 2/3. From the known dependence of the decrement on the amplitude of oscillations one can compute the drag coefficient. To this end the generalized mass μ was also determined in the experiments.

For the oscillations of the square plate in the direction perpendicular to the plane of the plate, the experimental results concerning the dependence of the decrement δ (the dots) and the resulting dependence of the drag coefficient c_x (the broken line) on the relative amplitude of oscillations $A/l = v_0/(\omega l)$ are shown in Fig.4b. The solid lines indicate the computational results. The theoretical dependence of c_x corresponds to the value of k for

$a/b = 1$ in Table 2. The decrement of the oscillations can be computed from the formula $\delta = D\,\mathrm{Sh}^{-1/2}$ with $D = B\,(\mathrm{Re})I\,(v_n)\,\rho R^3/\mu$, $\mathrm{Sh} = \omega R/v_0 = R/A$, and $R = l\ /2/$. Here we set $\rho = 1.293$ kg/m^3 (air) and $\mu = 11.74$ kg. The mass of the plate along with the associated mass μ of the air were determined experimentally. The decrement can be computed from the drag coefficient using the relation $D/k_x = 4\rho RS/(3\mu)$.

Analogous results for the rectangular plate are presented in Fig.4c. In this case the theoretical functions were obtained by linear interpolation from the values of k_x for $a/b = 3$ and $a/b = 4$ listed in Table 2 and for $\mu = 7.355$ kg.

In our experiment we realized angular oscillations of the square plate about the axis passing below the upper edge at a distance $h = 85$ mm. In Fig.4d the experimental dependence of the decrement of the oscillations on the amplitude of the angle θ is represented by the circles, and the computational dependence is represented by the solid line. The moment of inertia of the plate taking the associated mass of air into account amounts to $J = 4.627$ kg m^2. The values of the VIC's on the edges of the plate that are necessary for the computations were obtained using the superposition principle:

$$K_v = \left(1 - \frac{l}{2R}\right)K_{v1} + \frac{l}{2R}K_{v2}, \quad R = l - h$$

where K_{v1} and K_{v2} are the VIC's obtained in Sect.6 for the translational oscillations and the angular oscillations about the middle line. The characteristic dimension $a = l/2$ is used. Hence, if we set $v_0 = R\omega\theta$, then $D = B\,(\mathrm{Re})\rho a^3 R^2 I\,(v_n)/J$ and $\mathrm{Sh} = \omega a/v_0 = a/(R\theta)$ in the equation $\delta = D\,\mathrm{Sh}^{-1/2}$ for the decrement of the oscillations. Computations based on formula (1.3) yield $I\,(v_n) = 2.938$.

It is seen that in all cases the agreement between the computational and experimental data is satisfactory.

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